Journal of Fluids and Structures (1998) **12,** 1073–1085
Article No. fl980184

SPECIAL BRIEF NOTE

A SERIES IN $1/\sqrt{\text{Re}}$ TO REPRESENT THE STROUHAL–REYNOLDS NUMBER RELATIONSHIP OF THE CYLINDER WAKE

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(Received 15 June 1998 and in revised form 23 September 1998)

In this Brief Note, we show that shedding frequency data is well collapsed, over a large range of Re from 50 up to at least 140,000, by using a Strouhal number that depends upon an effective wake width, which includes not only the physical body diameter, but also a characteristic width of the separating shear layers. The use of this effective wake width also leads to a new formulation for the relationship between Strouhal number (S) versus Reynolds number (Re) for the cylinder wake, which may be expressed as an expansion in powers of $(1/\sqrt{\text{Re}})$:

$$\text{S} = \left(A + \frac{B}{\sqrt{\text{Re}}} + \frac{C}{\text{Re}} + \cdots \right).$$

Truncated two-term or three-term series have much less error-of-fit when compared with the traditional S–Re relationships commonly in use. A good test of any S–Re functional relationship is now made possible by comparison with Henderson's numerical data for *two-dimensional* laminar shedding, over a much larger range of Re (up to Re = 1000) than is possible to obtain experimentally. It seems significant that even a *two-term* fit, given by $\text{S} = 0{\cdot}2698 - 1{\cdot}0271/\sqrt{\text{Re}}$ has one order of magnitude less error than the traditional *three-term* fit. By using such $\sqrt{\text{Re}}$-formulae in both the laminar and 3-D wake turbulent regimes, we may accurately represent S–Re data over a large range of Re, although the validity of these representations at these higher Re needs further support. In summary, this Brief Note not only provides physical support for the use of such S–Re relationships as shown above, but also demonstrates that these formulations fit the data closer than traditional S–Re expressions.

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1. INTRODUCTION

FOLLOWING A NUMBER OF RECENT AND WELL-DEFINED INVESTIGATIONS into the wakes of bluff bodies, the question has arisen as to how the relation between the Strouhal number and the Reynolds number can best be represented, and what physical basis one can attribute to these representations. In the last decade, it has been shown that if parallel shedding can be induced (whereby the vortices in the wake and shed parallel to the cylinder axis) then the relation between Strouhal (S) and Reynolds (Re) numbers is continuous and remarkably repeatable, within the laminar vortex shedding regime. Recent developments in the understanding of wake vortex dynamics in general, and the techniques to induce parallel shedding, are reviewed in Williamson (1996), where a good agreement amongst recent results from different laboratories is shown. From his direct numerical simulations, Henderson (1997) has shown that the traditional formulation for the functional dependence between S and Re [see equation (4)] is inadequate, when extended to higher Re, as may be seen later in Figure 3. In this paper, we seek to formulate a new functional relationship which has more physical significance, and which fits the experimental/numerical data much closer, than the traditional forms. Although the present paper was originally in the form of an internal report (Williamson 1991), not otherwise published, the present paper grew out of the two authors' early collaboration, and has been triggered by the recent highly accurate two-dimensional computations of Henderson, referred to above.

A functional relationship for the original measurements by Strouhal (1878) of vortex-shedding frequency in the wake of a circular cylinder was put forward by Lord Rayleigh (1896, 1915). He suggested that a frequency parameter (now called the Strouhal number) could be expressed in terms of a Taylor's expansion involving a viscosity parameter (the inverse of what is now called the Reynolds number):

$$\mathrm{S} = \frac{fD}{U} = A + \frac{B}{\mathrm{Re}} + \frac{C}{\mathrm{Re}^2} + \cdots . \tag{1}$$

In this expression, f is the shedding frequency, D is the body diameter and U is the free-stream velocity. ($\mathrm{Re} = UD/\nu$. where ν is the kinematic viscosity). An interesting historical discussion of the use of equation (1), amongst other contributions by Lord Rayleigh, is contained in Rott (1992). A two-term truncated series for the S–Re relation was used by Roshko (1954) to represent the frequency of vortex shedding in different flow regimes, and these formulae have since been widely used.

The question of the degree of "bluffness" of a body was considered by Roshko (1955), who proposed a Strouhal number based on the physical scales of the near-wake formation to collapse frequency data from different body shapes. He suggested that the shedding frequency would scale on the wake width (L^*) rather than simply the body dimensions (D), and on a relevant velocity scale (U^*) for the vortex formation in the near wake, rather than simply on free stream velocity (U). He put forward a Strouhal number

$$\mathrm{S}^* = \frac{fL^*}{U^*} \approx \text{constant.} \tag{2}$$

The wake width L^* for different bodies was found using free-streamline theory, while the velocity scale was taken to be that velocity just outside the separation point (U_s) which, to good approximation, is calculated from the base pressure coefficient. (Subject to a boundary layer approximation, $\frac{1}{2}U_s^2$ is the flux of vorticity shed into the wake). This Strouhal number, and others which have been put forward, for example by Bearman (1967) and by Griffin (1981), resulted in a very reasonable collapse of the shedding-frequency data for many different bluff bodies. These results confirm the merit of considering the characteristic scales of wake formation rather than simply body dimensions (and free stream velocity). Such ideas also provide a basis for a functional relationship for S–Re measurements.

In this Brief Note, after defining such a Strouhal number S* which collapses well the frequency data over a large range of Re, we shall put forward a relationship between Strouhal–Reynolds numbers which involves an expansion in powers of $(1/\sqrt{\mathrm{Re}})$ rather than in powers of (1/Re). Since the first submission of this Note, Fey *et al.* (1998) have published a paper which shows visually that a plot of S versus $(1/\sqrt{\mathrm{Re}})$ for the laminar regime resembles a straight line. They have further usefully represented the variation of S–Re data from Re = 50 up to Re = 200 000, including many flow regimes, by a large set of tabulated straight line fits. The validation or physical motivation for these straight-line fits are not discussed, nor are the empirical fits compared to other possible S–Re relationships, some of which (for example Roshko number versus Reynolds number) also yield apparently straight line fits. The two-term expressions of Fey *et al.*, employed on the basis of visual inspection of S–$1/\sqrt{\mathrm{Re}}$ plots, might be regarded as the first two terms in a series expansion in $(1/\sqrt{\mathrm{Re}})$, which is accurate over a range of Reynolds numbers, but the coefficients were not chosen with a series expansion in mind. Nevertheless, their many curve fits (tabulated and plotted) to the S–Re data, over the several flow regimes up to Re = 200 000, provide a convenient source for such data, not hitherto available over this large range of Re.

In the present Note, we put forward the physical motivation for using such a series expansion in $(1/\sqrt{\mathrm{Re}})$ for the S–Re relationship, based on a characteristic wake width governing the shedding frequency. We also compare such truncated series with traditional S–Re expressions commonly used, and show that they yield closer fits to the measurements and simulations.

2. A SERIES IN $(1/\sqrt{\mathrm{Re}})$ TO REPRESENT S–Re MEASUREMENTS

Over the past 40 years, there have been a great many measurements which yielded different coefficients for S–Re relations for the laminar regime of shedding (typically within the range Re = 47–190). Such representations of the data generally followed the lead of Roshko (1954), who plotted (fD^2/ν) versus Re, and thus found coefficients A and B from a linear least-squares fit:

$$\mathrm{S} = A + \frac{B}{\mathrm{Re}}. \tag{3}$$

(Since that time, the parameter, $\mathrm{Ro} = (f\mathrm{D}^2/\nu) = \mathrm{S\,Re}$, has become known as the Roshko number.) Expression (3) represents the first two terms of Rayleigh's expansion (1). Following the work of Tritton (1959) and of Berger (1964), a three-term fit has also been widely used, which is typically derived by fitting a quadratic least-squares fit to a plot of Ro versus Re, so that

$$\mathrm{S} = A\mathrm{Re} + B + \frac{C}{\mathrm{Re}}, \tag{4}$$

where evidently coefficients A and B take on different values than in equation (3). In order to compare the accuracy of these "traditional" two- and three-term fits, these relations have been compared against experimental data, using the parallel-shedding measurements in Williamson (1988a, 1989). The two-term fit yields an error of around 10% at the lowest Re (see Figure 1), which is significant because data from different laboratories are now found to agree to within about 1%, as already discussed. On the other hand, the three-term relation fits the data to within about 2%, and is clearly preferable. The "error-of-fit" (i.e. the absolute value of the error averaged over all the data points; see the Table in Appendix A) for the two-term fit is found to be 0·0021 whereas for the three-term fit it is 0·0005.

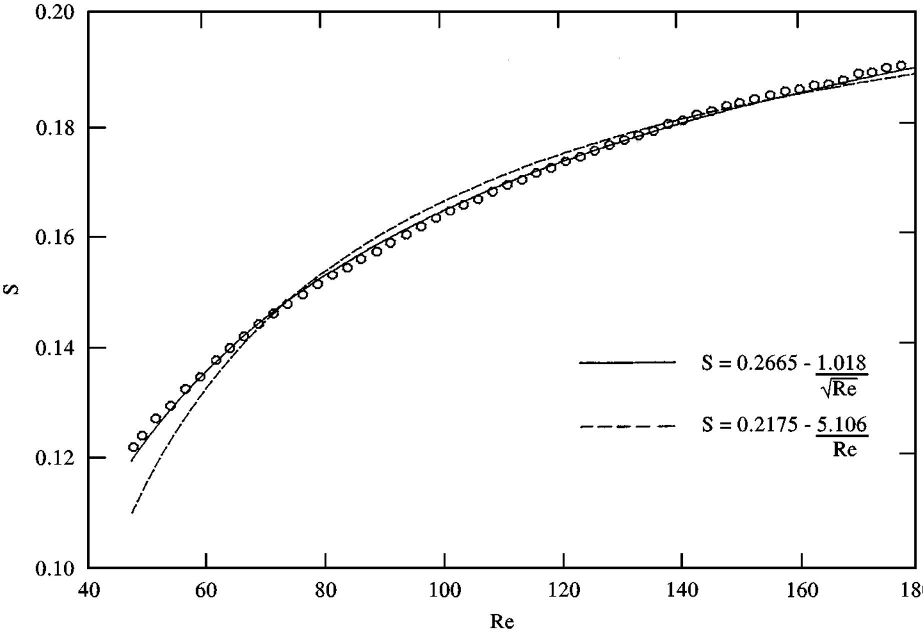

Figure 1. Comparison of the two-term "$\sqrt{\text{Re}}$-formula" versus the traditional two-term fit. The solid line is for the $\sqrt{\text{Re}}$-formula: $S = 0{\cdot}2665 - 1{\cdot}018/\sqrt{\text{Re}}$. The dashed line is for the traditional Re formula: $S = 0{\cdot}2175 - 5{\cdot}106/\text{Re}$.

It is possible to consider different length and velocity scales upon which the shedding frequency might depend. Bauer (1961) measured the vortex shedding frequencies behind flat plates parallel to the flow (whose cross-sections were bullet-shaped), for various values of the parameter C/D (chord-length/thickness). He normalized the Strouhal numbers by a characteristic length-scale equal to twice the boundary layer displacement thickness (δ^*) plus the plate (or "bullet") thickness (i.e., $D + 2\delta^*$). A subsequent study of Eisenlohr & Eckelmann (1988) for many different plates of large C/D ratios showed a good collapse of their frequency data when it was plotted using such a Strouhal number.

From a theoretical approach, Monkewitz & Nguyen (1987) have investigated the linear inviscid instability of a family of mean wake velocity profiles, on a locally parallel basis. Both the influence of the centreline velocity and the ratio of wake width to individual mixing layer thicknesses are explored. They consider different types of instability distributions in the downstream direction, involving convective (C) and absolute (A) instability. As a sequence going downstream from the body surface, two of the relevant distribution scenarios are C–A–C or simply A–C. For frequency selection, they deduce an "initial resonance criterion", which states that the frequency of the first downstream wake profile, where absolute instability is calculated, will dictate the global frequency of the wake. The agreement between this criterion and several bluff body measurements is remarkable. It is further supported by the experimental results in Unal & Rockwell (1988). Interestingly, there is a clear distinction between the frequency from the criterion, and the frequency computed to give the maximum spatial amplification [Pierrehumbert's (1984) criterion], which does not agree well with bluff body wake measurements.

The theoretical analysis of Monkewitz & Nguyen (1987) lends good support to the concept that the frequency scales with $(D + 2\delta)^{-1}$, where δ is a characteristic shear layer thickness. The length scale governing the frequency in the theory is twice the wake half-width ($y_{1/2}$), which naturally includes not only the part of the velocity profile with the most deficit but also includes a characteristic measure of the shear layer thickness; the global frequency scales with $(2y_{1/2})^{-1}$. As expected, if our shear layer becomes larger compared with diameter (i.e. δ/D larger), the frequency correspondingly diminishes. However, one must note that the overall "shape" of the profile changes (if centreline velocity is kept constant, the "shape" is defined by parameter $N \sim 1/\delta_\omega$, in Monkewitz & Nguyen), which is expected to further diminish the instability frequency in the theory. Monkewitz & Nguyen show that over a range of N shape factors (relevant to our data here), and for constant $y_{1/2}$, the "initial resonance criterion" selects global wake frequencies which remain approximately the same. The conclusion is that the theory supports the concept that the selected frequency in the wake will reasonably scale with $(D + 2\delta)^{-1}$, measured close to the body.

In the present Note concerning the cylinder wake, consistently with the above discussion, we shall use the concept that the frequency will scale with $(D + 2\delta)^{-1}$ where δ is a characteristic shear layer thickness (without specifying it precisely at this point). Following Roshko (1955), we formulate a Strouhal number S^* which should be nearly constant over a wide range of Reynolds numbers. In equation (2), $L^* = (D + 2\delta)$, and $U^* = (U_s)$ is the velocity measured at separation, giving

$$\mathrm{S}^* = \left(\frac{fD}{U}\right)\left(\frac{U}{U_s}\right)\left(1 + \frac{2\delta}{D}\right). \tag{5}$$

It can be shown (Roshko 1955) that U_s is related to the base pressure coefficient (to good approximation) by

$$\frac{U_s}{U} = \sqrt{1 - C_{pb}}, \tag{6}$$

giving an expression for the Strouhal number as

$$\mathrm{S} = \mathrm{S}^* \sqrt{1 - C_{pb}}\left(1 + \frac{2\delta}{D}\right)^{-1}. \tag{7}$$

The base pressure parameter $\sqrt{1 - C_{pb}}$ varies only by around 10% for a circular cylinder over the laminar range of Re, and is very well represented by an expansion in powers of $1/\sqrt{\mathrm{Re}}$:

$$\sqrt{1 - C_{pb}} = C' - D'/\sqrt{\mathrm{Re}}, \tag{8}$$

where $C' = 1{\cdot}548$ and $D' = 2{\cdot}328$ using the measurements of Williamson & Roshko (1990).

Experimental support for the use of the Strouhal number S^* in equation (7) comes from Figure 2(a), where we have plotted S versus $\sqrt{1 - C_{pb}}(1 + 2\delta/D)^{-1}$, and note that the data lie closely along a straight line, whose gradient yields the best-fit value for $\mathrm{S}^* = 0{\cdot}176$. One should note that the available data from the literature for this plot includes the range of Re from 55 up to 140 000. A plot of calculated S^* values versus Re in Figure 2(b) gives another indication of the constancy of S^* over this large range of Re, and thus suggests it is a reasonable means to collapse frequency data for this body.

The separating shear layer thickness δ will depend on the growth of the boundary layer on the forward part of a cylinder which, subject to a boundary layer approximation, gives $\delta/D \sim 1/\sqrt{\mathrm{Re}}$. This relationship was earlier assumed by Bloor (1964), who used it to show that the shear layer instability frequency scaled approximately with $\sqrt{\mathrm{Re}}$. Measured values of shear layer vorticity thickness δ_ω/D for Re up to 1200 [where δ_ω is defined as the

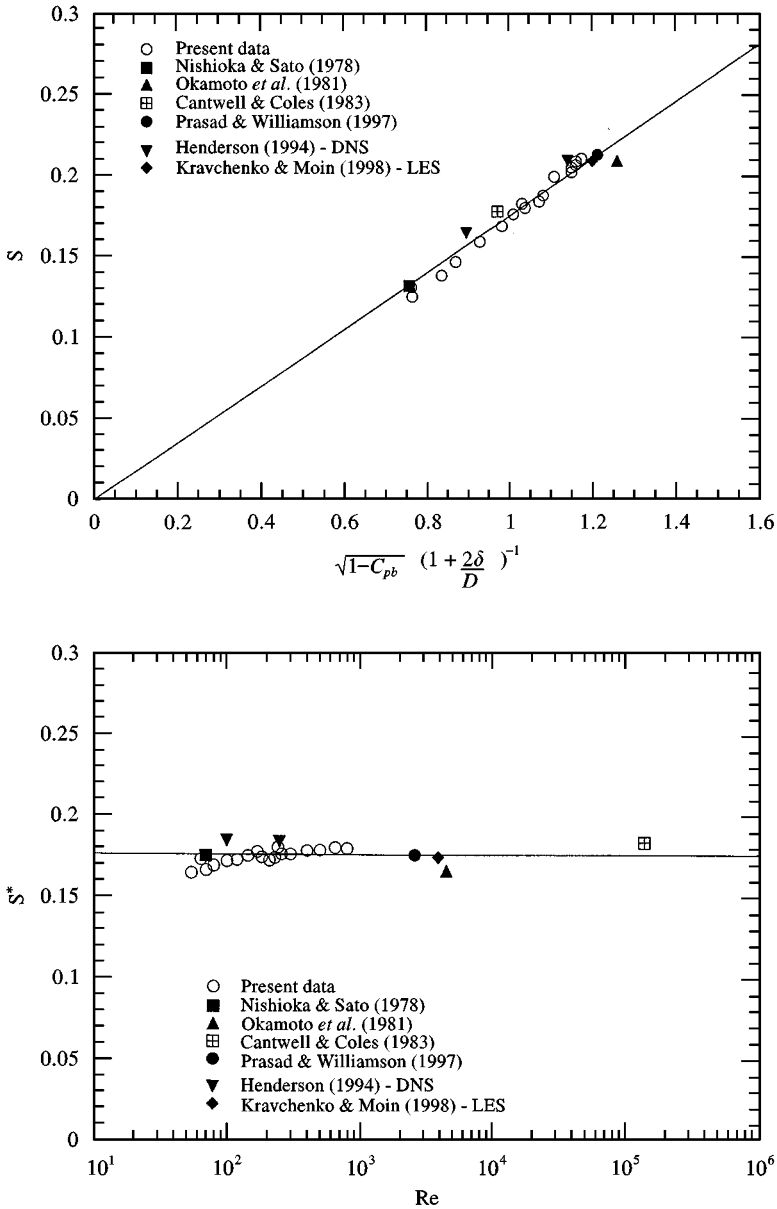

Figure 2. Collapse of frequency data using a wake length scale involving the body diameter plus the shear layer thickness. The separated shear layer thickness δ used in these data is $\delta_\omega/2$, evaluated at $x/D = 1{\cdot}0$ in the near wake.

difference in velocities across the layer divided by the maximum slope of the velocity profile; see Huerre & Monkewitz (1985)] evaluated at a station that is one diameter downstream of the cylinder axis, were plotted against Re (unpublished work by Williamson and Brown, Caltech 1988), and showed close comparison with the formula

$$\frac{\delta_\omega}{D} = \frac{4{\cdot}217}{\sqrt{\mathrm{Re}}}. \tag{9}$$

By plotting Strouhal number versus δ_ω/D using equation (9), it was visually observed that the data lie very closely along a straight line given by

$$\mathrm{S} = 0{\cdot}267 - 0{\cdot}241(\delta_\omega/D). \tag{10}$$

It is this result that first suggested to the authors the present study of the S–Re functional relationship, because it immediately suggests that an accurate relationship for S–Re involving $\sqrt{\mathrm{Re}}$ is readily to be found. It is expected that other measures of shear layer thickness (for example displacement thickness δ^* or momentum thickness θ) will also scale similarly with $(1/\sqrt{\mathrm{Re}})$, and will be smaller than δ_ω. In the ensuing discussion, it is not clear that one may precisely specify (ahead of time) a particular measure of shear layer thickness (for example δ^*, δ_ω or θ) which will govern the shedding frequency. The use of δ in the following equations is taken to mean a measure of thickness (proportional to δ^* for example) that is expected to scale with $(1/\sqrt{\mathrm{Re}})$. A relationship for S–Re involving $\sqrt{\mathrm{Re}}$ follows from equation (7), where we can expand $(1 + 2\delta/D)^{-1}$ for small δ/D:

$$\mathrm{S} = \mathrm{S}^* \sqrt{1 - C_{pb}} \left(1 - \frac{2\delta}{D} + \mathcal{O}\left(\frac{2\delta}{D}\right)^2 - \cdots \right), \tag{11}$$

and then use the relation $\delta/D \sim 1/\sqrt{\mathrm{Re}}$ and equation (8) to give

$$\mathrm{S} = \left(A + \frac{B}{\sqrt{\mathrm{Re}}} + \frac{C}{\mathrm{Re}} + \cdots \right). \tag{12}$$

Clearly, one must question whether δ/D is indeed typically small, and this depends on which measure of shear layer thickness is taken. The choice of displacement thickness, as in the work of Bauer (1961), is of the order of 5 times smaller than the vorticity thickness for the shear layer profiles found here, and at Re $\sim$ 100, would yield a third term in the expansion of equation (11) which is of order 4% compared to unity, providing some support for the expansion in terms of small δ/D. By truncating the series (12) to only three terms, we find a least-squares fit for the laminar shedding regime, using the parallel shedding data from Williamson (1988a, 1989):

$$\mathrm{S} = \left(0{\cdot}285 - \frac{1{\cdot}390}{\sqrt{\mathrm{Re}}} + \frac{1{\cdot}806}{\mathrm{Re}}\right), \tag{13}$$

from which it can be noted that, at Re $\sim$ 100, the third term is of order 6% of the first term.

The expansion (12) is obviously different to that proposed by Lord Rayleigh in that it involves powers of $(1/\sqrt{\mathrm{Re}})$ rather than powers of (1/Re). Such an expansion was also suggested to the first author by Professors Nicholas Rott and Milton Van Dyke of Stanford (private communication, 1991), on the basis that in the bluff-body wake there is a role played by the separating boundary layers, for which we should expect a $\sqrt{\mathrm{Re}}$ type of scaling. Generally, such an expansion is made away from the point at $\mathrm{Re}^{-1/2} = 0$, although here this is not possible. A single mode of shedding, and thereby a single curve to relate S–Re, cannot be expected over a large range of Re, because different instabilities appear as Re is increased. This matter is discussed further in Section 4.

It should be noted that the three-term expression (13) has an average error-of-fit of only 0·0002, and is distinctly more accurate than existing traditional fits (see Table 1). [A more simple formula involving only two terms of the expansion (12) may also be written:

$$\mathrm{S} = A + \frac{B}{\sqrt{\mathrm{Re}}}. \tag{14}$$

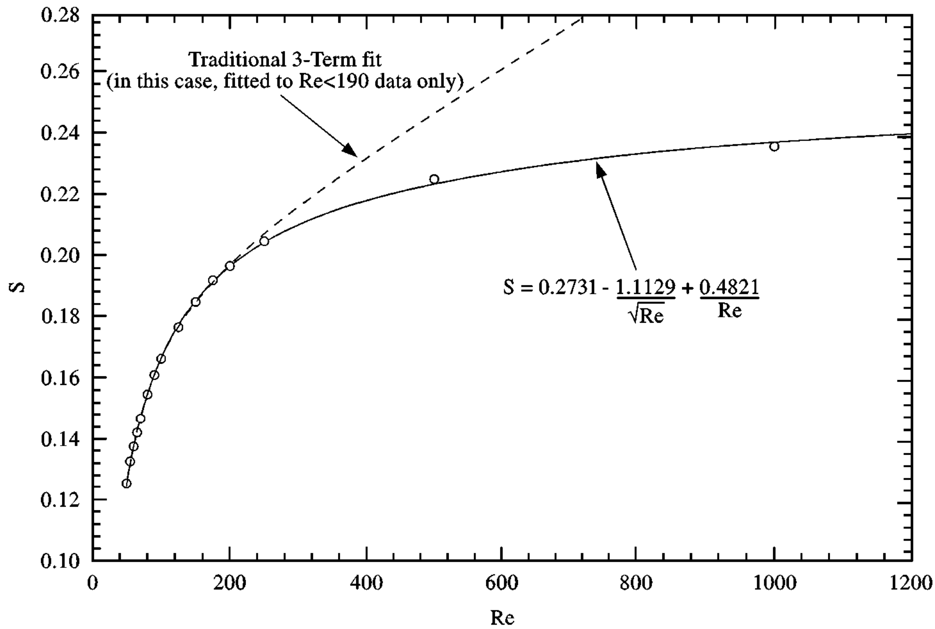

Figure 3. Fit of $\sqrt{\mathrm{Re}}$-formula over a very large laminar shedding regime, made possible by numerical simulation (Henderson 1997). An excellent fit of numerical S–Re data up to Re = 1 000 is made by a three-term $\sqrt{\mathrm{Re}}$-formula. [Note: The traditional three-term equation (4), when fitted to the data up to Re = 190, is clearly not a good representation of the Strouhal numbers when Re is increased further, as demonstrated by the divergence of the dashed line from Henderson's data. This point is expounded in Henderson 1997]

At low Reynolds numbers, this function might be seen more as a curve fit than two terms in an expansion but such a relation can be fit to the parallel-shedding data:

$$S = 0{\cdot}2665 - \frac{1{\cdot}018}{\sqrt{\mathrm{Re}}}, \tag{15}$$

and may be compared with the traditional two-term fit to the same data:

$$S = 0{\cdot}2175 - \frac{5{\cdot}106}{\mathrm{Re}}. \tag{16}$$

Figure 1 shows that the "$\sqrt{\mathrm{Re}}$-formula" (15) lies much closer to the experimental data than the traditional formula (16), giving an average error-of-fit of 0·0006 for the $\sqrt{\mathrm{Re}}$ formula as compared with 0·0021 for the Re formula (as quoted earlier). In fact, the error for the two-term $\sqrt{\mathrm{Re}}$-formula is comparable with the error for a traditional three-term fit, equation (4).]

An interesting question arises, however, as to whether the three-term expression (13) might be appropriately thought of as an asymptotic series or also as another curve fit. Following the two-dimensional computations of Henderson (1997), we are now in a fortunate position to investigate the use of these formulae in a highly extended laminar regime, up to Re = 1000 (see Figure 3). The S–Re data is kindly made available by Henderson (1998). Henderson's simulations extend the laminar regime far beyond the critical Re (190), where the experimental near-wake flow would otherwise become three-dimensionally unstable, and would therefore have different near-wake dynamics. These computations provide a good test of the use of different functional S–Re relationships. The use of the three-term series in $1/\sqrt{\mathrm{Re}}$ (see Table 1),

$$\mathrm{S} = \left(0{\cdot}2731 - \frac{1{\cdot}1129}{\sqrt{\mathrm{Re}}} + \frac{0{\cdot}4821}{\mathrm{Re}}\right), \tag{17}$$

yields excellent precision in representing the data (average error-of-fit = 0·0004), as shown in Figure 3. However, even a comparison of a two-term $\sqrt{\mathrm{Re}}$-formula versus the traditional three-term equation (4), both evaluated for Henderson's data, demonstrates that the $\sqrt{\mathrm{Re}}$-formula fits the numerical data far better than the traditional equation (4). Not only is the error-of-fit (0·0006) an order of magnitude less than for the traditional equation (0·0046), but this is achieved with two terms in the equation rather than three. The above results lend support towards validating the practical use of the expansion in terms of $(1/\sqrt{\mathrm{Re}})$.

3. USE OF EXPANSIONS IN $(1/\sqrt{\mathrm{Re}})$ AT HIGHER REYNOLDS NUMBERS

The wake of a cylinder becomes 3-D turbulent to small scales (smaller than the primary wavelength) at roughly Re = 190, after which the primary shedding vortices become unstable to a spanwise waviness (termed as mode A), corresponding with a Strouhal number curve A in Figure 4 (taken from Williamson 1988b). Between Re = 230 and 260, there is a transition to a mode B 3-D shedding, which involves finer-scale streamwise vortices appearing in the near wake, and corresponds with curve B in Figure 4. This curve continues up to higher Re, at least beyond 1200. If we apply the $\sqrt{\mathrm{Re}}$ formulation to the

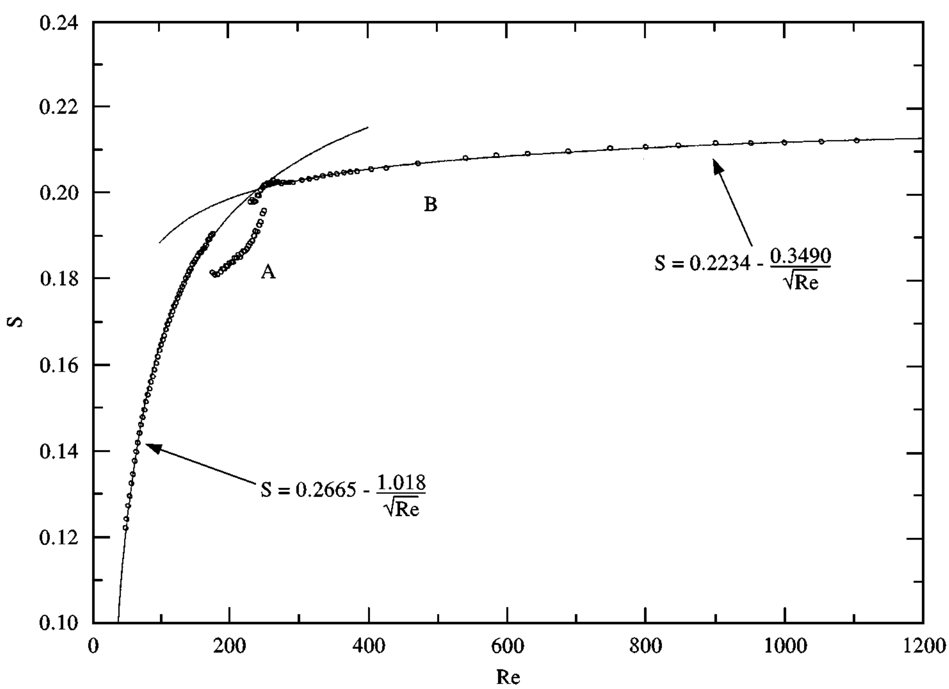

Figure 4. Fit of $\sqrt{\mathrm{Re}}$ formulae to the laminar and 3-D turbulent regimes. In this plot the S–Re data is essentially represented very well by two $\sqrt{\mathrm{Re}}$ formulae up to (at least) Re = 1200 (with the exception of mode A in the transition regime. Of particular interest is the very sharp change in S–Re functional relationship at Re close to 260.

frequency data for mode B, we find

$$S = 0{\cdot}2234 - \frac{0{\cdot}3490}{\sqrt{\mathrm{Re}}}, \tag{18}$$

giving an average error-of-fit of only 0·0005. The $\sqrt{\mathrm{Re}}$ formulae in equations (18) and (15) are shown together with the measured data (Williamson 1988b) in Figure 4 for the laminar and 3-D turbulent regimes. Interestingly, it can be seen that the intersection of the two frequency curves occurs near Re = 260, where there is a sharp change in the functional relationship of S–Re, and where mode B (3-D) vortex shedding starts to dominate. Although the three-dimensional near wake vorticity dynamics for Re > 250 affect the shedding frequency, the characteristic length scale for the governing frequency again appears to be well described by a $(1/\sqrt{\mathrm{Re}})$ dependence.

4. CONCLUDING REMARKS

In summary, we have formulated a new functional relationship between Strouhal number and Reynolds number for the cylinder wake, which may be written as an expansion in $(1/\sqrt{\mathrm{Re}})$:

$$S = \left(A + \frac{B}{\sqrt{\mathrm{Re}}} + \frac{C}{\mathrm{Re}} + \cdots\right).$$

The motivation for such an expansion comes from the use of a wake length scale, upon which the frequency depends, which incorporates both the physical dimensions of the bluff body plus the thickness of the separating shear layers. The Strouhal number S*, which incorporates this wake length scale, collapses well the shedding frequency data from various investigations encompassing a large range of Re from 50 up to 10^5.

The recent extension of the "numerical" laminar regime by Henderson (1997), up to Re = 1000, by the use of two-dimensional computations, provides a good test of such functional relationships. An excellent representation of the simulation data involves a truncated series of three terms:

$$S = \left(0{\cdot}2731 - \frac{1{\cdot}1129}{\sqrt{\mathrm{Re}}} + \frac{0{\cdot}4821}{\mathrm{Re}}\right). \tag{17}$$

Interestingly, it is found that even a two-term fit yields a better fit to the data than the three-term traditional fit (an order of magnitude less error-of-fit).

A physical interpretation of the $\sqrt{\mathrm{Re}}$-formula is that the constant term (A) is due to the size or physical shape of the body itself, while the following terms in powers of $(1/\sqrt{\mathrm{Re}})$ are associated with the shear layer thickness. One might then expect, for very large Re, that the Strouhal number will reach a saturation value equal to the constant A. However, in experiment and even in two-dimensional computations, the separating shear layers themselves become unstable at sufficiently high Re > 1200 [see experiments of Bloor (1964) and the 2-D-computations of Braza *et al.* (1990)], and the constant A will *not* represent a "saturation" value of Strouhal number. This is because the shear layer instability has an effect on the vortex dynamics in the near wake. One should note that in experiment, the Strouhal number actually *decreases* in a broad range from Re $\sim$ 2000 up to around 10^5, suggesting that the turbulent shear layer instability effectively widens the wake width, L^*, as one may also deduce from the pressure measurements of Schiller & Linke (1933).

Although the preceding results have been applied principally to the wake of a cylinder, the approach could well be applicable to a class of bluff body wakes, particularly to cases where the separation point is fixed, for example the D-Section cylinder (Huang & Brown 1998).

ACKNOWLEDGEMENTS

Particular thanks are due to Nicholas Rott and Milton Van Dyke (Stanford University) for triggering the writing of this paper (in the form of an internal report) in 1991. Following a study of the early contributions to the subject by Lord Rayleigh (see Rott 1992), they suggested to the first author a $(1/\sqrt{Re})$ expansion for S–Re measurements, which corresponded well with such curve fits made by the first author at Caltech in 1989. The author would like to acknowledge the enthusiastic help from Kristen Gledhill (1991), Anil Prasad (1991), and Raghu Govardhan (1998) at Cornell for calculating most of the curve fits. The support from the Ocean Engineering Division of the O.N.R., monitored by Tom Swean, is gratefully acknowledged (O.N.R. Contract No. N00014-95-1-0332).

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APPENDIX A: COMPARISON OF S–Re FUNCTIONAL CURVE FITS

ε = absolute value of ($S_{measured} - S_{fit}$) averaged over all the data points.

A.1. "EXPERIMENTAL" LAMINAR REGIME, Re ~ 49–180 WILLIAMSON (1988a)

Description	Function	Coefficients	ε
3-term traditional	$S = A\text{Re} + B + C/\text{Re}$	$A = 1{\cdot}600\times10^{-4}$ $B = 0{\cdot}1816$ $C = -3{\cdot}3265$	0·0005
2-term-traditional	$S = A + B/\text{Re}$	$A = 0{\cdot}2175$ $B = -5{\cdot}1064$	0·0021
3-term expansion in $(1/\sqrt{\text{Re}})$	$S = A + B/\sqrt{\text{Re}} + C/\text{Re}$	$A = 0{\cdot}2850$ $B = -1{\cdot}3897$ $C = 1{\cdot}8061$	0·0002
2-term "$\sqrt{\text{Re}}$-formula"	$S = A + B/\sqrt{\text{Re}}$	$A = 0{\cdot}2665$ $B = -1{\cdot}0175$	0·0006

A.2. "NUMERICAL" LAMINAR REGIME, Re ~ 47–1000 DNS COMPUTATIONS OF HENDERSON (1997)

Description	Function	Coefficients	ε
3-term traditional	$S = A\text{Re} + B + C/\text{Re}$	$A = 1{\cdot}685 \times 10^{-5}$ $B = 0{\cdot}2263$ $C = -5{\cdot}8556$	0·0046
3-term expansion in $(1/\sqrt{\text{Re}})$	$S = A + B/\sqrt{\text{Re}} + C/\text{Re}$	$A = 0{\cdot}2731$ $B = -1{\cdot}1129$ $C = 0{\cdot}4821$	0·0005
2-term "$\sqrt{\text{Re}}$-formula"	$S = A + B/\sqrt{\text{e}}$	$A = 0{\cdot}2698$ $B = -1{\cdot}0272$	0·0006